

Claims

1. A method of fragile watermarking a digital image,
including the steps of extracting from the image a portion
5 thereof A and generating at least a first ill-conditioned
operator related to values extracted from the portion A .
2. A method of fragile watermarking according to claim 1
wherein the ill-conditioned operator is generated by
10 altering a value to increase the operator's condition
number.
3. A method of fragile watermarking according to claim 1
or 2, comprising the step of replacing a non-zero singular
15 value of a singular value matrix S_A of an image or portion
thereof A , with a solution to a linear equation comprising
the ill-conditioned operator.
4. A method of fragile watermarking according to claim 3,
20 wherein the non-zero singular value to be replaced is the
smallest non-zero singular value $S_r(A)$ in a singular value
matrix S_A of rank r .
5. A method of fragile watermarking according to any one
25 of the preceding claims, wherein a non-zero singular value
of a singular value matrix S_W of a watermark pattern or
portion thereof W is replaced, such that said replacement
increases the condition number of the singular value matrix
 S_W of the watermark pattern or portion thereof W .
- 30 6. A method of fragile watermarking according to claim 5,
wherein the non-zero singular value to be replaced is the
smallest non-zero singular value $S_t(W)$ in a singular value
matrix S_W of rank t .

7. A method of fragile watermarking according to any one of the preceding claims, wherein the step of calculating a replacement non-zero singular value of singular value matrix S_w of a watermark or portion thereof W comprises
 5 calculating substantially the following equation part:

$$s_t(W) = \varepsilon,$$

where ε is a small positive real number that increases the condition number of the singular value matrix S_w .

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8. A method of fragile watermarking according to any one of the preceding claims, wherein the step of generating at least a first ill-conditioned operator comprises calculating substantially the following equation part:

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$$B = \hat{A}\hat{W},$$

where \hat{W} is substantially constructed according to $\hat{W} = U_w \hat{S}_w V_w^T$, \hat{S}_w comprising at least one altered singular value $s_t(W) = \varepsilon$, and such that B forms a parametric family of matrices $B(\hat{S}_r) = \hat{A}(\hat{S}_r)\hat{W}$ for possible values of $\hat{S}_r(A)$.

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9. A method of fragile watermarking according to claim 8, wherein $s_r(A)$ is determined by an L_2 -norm solution of the

least squares problem $\min_{x \in \mathbb{R}^p} \|Bx - b\|_2^2$ to equal the square

of a predefined key N of predetermined value, where b is an
 25 arbitrary vector.

10. A method of fragile watermarking according to any one of the preceding claims, wherein the step of calculating the replacement non-zero singular value of singular value matrix A comprises calculating substantially the following
 30 equation part:

$$\min_{\hat{s}_r(A)} \left\{ \sum_{i=1}^q \left(u_{B_i}^T b / s_i(B(\hat{s}_r)) \right)^2 - N^2 \right\},$$

where u_{B_i} is the i -th column of the matrix formed with the right singular vectors of B .

- 5 11. A method of fragile watermarking according to claim 10, wherein $\hat{s}_r(A)$ further satisfies

$\hat{s}_r(A) = \overline{s}_r(A) \in [\max(\text{eps}, s_r(A) - \delta), s_r(A) + \delta] = [H_0, H_1]$, where δ is a distortion control and eps is machine precision, such that the step of calculating the replacement non-zero
 10 singular value comprises calculating substantially the following equation part:

$$\min_{\hat{s}_r \in [H_0, H_1]} \left\{ \sum_{i=1}^q \left(u_{B_i}^T b / s_i(B(\hat{s}_r)) \right)^2 - N^2 \right\},$$

with all terms as defined herein.

- 15 12. A method of fragile watermarking according to any one of claims 9 to 11, wherein vector b is related to at least a first parameter derived from a portion of an image I other than A .

- 20 13. A method of fragile watermarking according to claim 12, wherein for a sequential watermarking process comprising the watermarking of portion $A^{(k)}$ after the watermarking of portion $A^{(k-1)}$, $k=1, \dots, L$ of L portions, then the step of calculating $b^{(k)}$ for portion $A^{(k)}$ comprises
 25 calculating substantially the following equation part:

$$b^{(k)} = \begin{cases} A^{(k)} Z^{(k)} & \text{for } k = 1 \\ A^{(k-1)} Z^{(k)} & \text{else} \end{cases},$$

where $Z^{(k)}$ is a pseudo-random binary vector.

14. A method of fragile watermarking according to any one of the preceding claims, wherein the step of calculating the watermarked image or portion thereof \hat{A} comprises calculating substantially the following equation part:

$$5 \quad \hat{A} = U_A \hat{S}_A V_A^T$$

where \hat{S}_A comprises at least one replaced singular value, U_A and V_A being left and right singular matrices.

15. A method of fragile watermarking according to any one of the preceding claims, wherein a watermark pattern or portion thereof W is generated by a pseudo-random generator seeded by a key K of predetermined value.

16. A method of fragile watermarking according to claim 15, wherein the values of key K and key N are related.

17. A method of fragile watermarking according to either one of claims 15 and 16, wherein the a watermark pattern or portion thereof W is generated by a pseudo-random generator seeded by a key K of predetermined value, combined with either a single or repeated instance of a logo.

18. A method of fragile watermarking according to any one of the preceding claims, comprising the following steps;

25 i. generating a K -dependent watermark pattern W from Ω , or recalling a pre-existing one;

 ii. constructing a parametric family of matrices $B(\hat{S}_r)$;

 iii. estimating a unique parameter $\bar{S}_r(A)$, that minimizes the expression

$$30 \quad \min_{\hat{S}_r} \left\{ \sum_{i=1}^q \left(u_{B_i}^T b / s_i(B(\hat{S}_r)) \right)^2 - N^2 \right\}; \text{ and}$$

- iv. estimating the watermarked block $\hat{A} = U_A \hat{S}_A V_A^T$ by
 setting $\hat{S} = \text{diag}(s_1(A), \dots, s_{r-1}(A), \bar{s}_r(A))$.

19. A method of fragile watermarking according to any one
 5 of claims 1 to 17, comprising the following steps;
 i. generating a K -dependent watermark pattern W from
 Ω , or recalling a pre-existing one;
 ii. constructing a parametric family of matrices $B(\hat{S}_r)$;
 iii. estimating a unique parameter
 10 $\bar{s}_r(A) \in [\max(\text{eps}, s_r(A) - \delta), s_r(A) + \delta] = [H_0, H_1]$, that
 minimizes the expression:

$$\min_{\hat{S}_r \in [H_0, H_1]} \left\{ \sum_{i=1}^q (u_{B_i}^T b / s_i(B(\hat{S}_r)))^2 - N^2 \right\}; \text{ and}$$

- iv. estimating the watermarked block $\hat{A} = U_A \hat{S}_A V_A^T$ by
 setting $\hat{S} = \text{diag}(s_1(A), \dots, s_{r-1}(A), \bar{s}_r(A))$.

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20. A method of verifying a fragile watermark,
 characterised by the step of generating at least a first
 ill-conditioned operator by altering a value to increase
 its condition number, said ill-conditioned operator being
 20 related to values extracted from a received image or
 portion thereof A^* .

21. A method of verifying a fragile watermark according to
 claim 20, characterised by the step of calculating a
 25 solution to the least squares problem $\min_{x \in \mathbb{R}^p} \|B^* x - b\|_2^2$ where
 $B^* = A^* \hat{W}$.

22. A method of verifying a fragile watermark according to
 either one of claims 20 and 21, wherein a positive square-

root N^* of the L_2 -norm solution of the least squares

problem $\min_{x \in \mathbb{R}^p} \|B^*x - b\|_2^2$ is compared with key N ; and

the received image or portion thereof A^* comprising
the fragile watermark is declared authentic if $|N^* - N| \leq \tau$,

5 where τ is a threshold value.

23. A method of verifying a fragile watermark according to
any one of claims 20 to 22, wherein the step of calculating
value N^* comprises calculating substantially the following
10 equation part:

$$(N^*)^2 = \sum_{i=1}^n \left(u_{B_i}^T b / s_i(B^*) \right)^2;$$

N^* is compared with key N ; and

the received image or portion thereof A^* comprising
the fragile watermark is declared authentic if $|N^* - N| \leq \tau$,

15 where τ is a threshold value.

24. Apparatus for fragile watermarking of an image in
accordance with a method of any one of claims 1 to 19,
comprising generating means for generating at least a first
20 ill-conditioned operator, said ill-conditioned operator
being related to values extracted from an image or portion
thereof A .

25. Apparatus for validating a fragile watermarked image
25 in accordance with a method of any one of claims 20 to 23,
and comprising;

generating means for generating at least a first ill-
conditioned operator by altering a value to increase its
condition number, said ill-conditioned operator being

related to values extracted from a received image or
portion thereof A' .